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Methods for the Nonperturbative Approximation of Form Factors and Scattering Amplitudes[1]

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Abstract. Methods are described for the nonperturbative calculation of wave functions and scattering amplitudes in light-cone quantization. Form factors are computed from the boost-invariant wave functions, which appear as coefficients in a Fock-state expansion of the field-theoretic eigenstate. A technique is proposed for calculating scattering amplitudes from matrix elements of a T operator between such composite-particle eigenstates.

INTRODUCTION

To benefit from the recent progress on the calculation of field-theoretic bound states in light-cone quantization [1,2], we explore methods by which form factors and scattering amplitudes can be extracted nonperturbatively. In the case of form factors, this is relatively straightforward; well-known formulas [3] yield the form factors as overlap integrals of Fock-state wave functions. For scattering amplitudes, the way is less certain. One possible method [4] is discussed briefly here. Others have been considered by Kröger [5], Ji and Surya [6], and Fuda [7].

The formulations given are in terms of light-cone coordinates [8,1], where $x^+ \equiv t + z$ plays the role of time and the conjugate variable $p^- \equiv E - p_z$ is the light-cone energy. The light-cone three-momentum is $\underline{p} = (p^+ \equiv E + p_z, \mathbf{p}_\perp)$. An eigenstate $|P, \sigma\rangle$ of the light-cone Hamiltonian operators $\mathcal{P}^\pm$, $\mathcal{P}_\perp$ and helicity σ is written as a Fock-state expansion

$$|P, \sigma\rangle = \sum_n \int [dx][d^2k_\perp] \psi^{(n)}_{P,\sigma}(x, \mathbf{k}_\perp) |n : \underline{p}_i\rangle \,, \tag{1}$$

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with

$$\int [dx][d^2k_\perp] = \int \delta(1-\sum_i x_i)\prod_i \frac{dx_i}{\sqrt{x_i}} 16\pi^3\delta(\sum_i \mathbf{k}_{\perp i})\prod_i \frac{d^2k_{\perp i}}{16\pi^3} \tag{2}$$

and where the $\psi^{(n)}$ are wave functions for n particles, $x_i \equiv p_i^+/P^+$ are longitudinal momentum fractions, and $\mathbf{k}_{\perp i} = \mathbf{p}_{\perp i} - x_i\mathbf{P}_\perp$ are relative transverse momenta. Use of light-cone coordinates brings several advantages, including boost invariance of the wave functions.

The eigenvalue problem $\mathcal{P}|P,\sigma\rangle = P|P,\sigma\rangle$ for fixed σ determines the wave functions as solutions of a coupled set of integral equations. A method frequently applied to these equations is discrete light-cone quantization (DLCQ) [9,1], which approximates the integrals by the trapezoidal rule and computes the wave functions on an equally spaced momentum grid. Any bound-state property can then, in principle, be calculated from these wave functions. The grid is parameterized by a longitudinal resolution K and transverse resolution $N_\perp$, such that longitudinal momentum fractions are multiples of $1/K$ and transverse momenta have as many as $2N_\perp + 1$ values in each direction. The value of $N_\perp$ is associated with a cutoff Λ^2 on the invariant mass of each constituent and with the choice of transverse momentum scale $\pi/L_\perp$.

FORM FACTORS

For a spin-1/2 fermion, the two form factors can be obtained from matrix elements of the plus component of the electromagnetic current J

$$F_1(Q^2) = \frac{1}{2}\langle P+Q,\sigma|J^+(0)/P^+|P,\sigma\rangle\,, \tag{3}$$

$$-\left(\frac{Q_x - iQ_y}{2M}\right)F_2(Q^2) = \frac{1}{4\sigma}\langle P+Q,\sigma|J^+(0)/P^+|P,-\sigma\rangle\,. \tag{4}$$

These can be reduced to overlap integrals [3]

$$F_1(Q^2) = \sum_n\sum_j e_j \int [dx][d^2k_\perp]\psi^{(n)*}_{P+Q,1/2}(x,\mathbf{k}'_\perp)\psi^{(n)}_{P,1/2}(x,\mathbf{k}_\perp)\,, \tag{5}$$

$$-\left(\frac{Q_x - iQ_y}{2M}\right)F_2(Q^2) = \sum_n\sum_j e_j \int [dx][d^2k_\perp]\psi^{(n)*}_{P+Q,1/2}(x,\mathbf{k}'_\perp)\psi^{(n)}_{P,-1/2}(x,\mathbf{k}_\perp)\,, \tag{6}$$

in the frame where the photon momentum Q is written $(0, 2Q\cdot P/P^+, \mathbf{Q}_\perp)$ and

$$\mathbf{k}'_{\perp i} = \begin{cases} \mathbf{k}_{\perp i} - x_i\mathbf{Q}_\perp\,, & i \neq j \\ \mathbf{k}_{\perp j} + (1-x_j)\mathbf{Q}_\perp\,, & i = j\,. \end{cases} \tag{7}$$

For the model studied by Brodsky, Hiller, and McCartor [2], an explicit calculation of F_1 has been done [10]. In this model, a bare fermion acts as a source and sink for bosons of mass μ. The lowest massive eigenstate is a fermion dressed by a boson cloud. The theory is regulated by a Pauli–Villars boson [11] with an imaginary coupling, and renormalized by fits of physical quantities to "data." Because no spin-flip interactions are included, F_2 is zero. Results for F_1 are shown in Fig. 1. The large-momentum-transfer value of F_1 is the bare fermion probability and therefore is not zero.

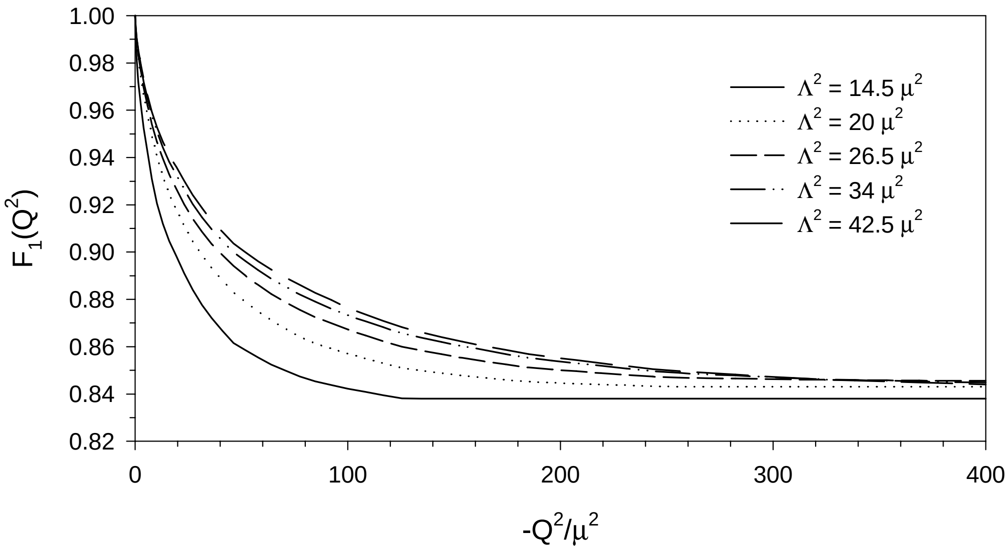

FIGURE 1. The form factor F_1 for fixed longitudinal resolution $K = 9$ and transverse scale $L_\perp = 2\pi/\mu$, and for a particular set of model parameters. Various cutoffs Λ^2 are considered, with the transverse resolution $N_\perp$ ranging from 5 to 9.

SCATTERING AMPLITUDES

The center-of-mass cross section for two-body scattering ($A + B \to C + D$) is [12]

$$\frac{d\sigma}{d\Omega}_{\rm cm} = \frac{1}{2E_A 2E_B v_{\rm rel}} \frac{|\vec{p}_C||\mathcal{M}_{fi}|^2}{16\pi^2 E_{\rm cm}}\,, \tag{8}$$

where $\mathcal{M}_{fi}$ is the invariant amplitude obtained from the S matrix

$$S_{fi} = \langle f|i\rangle + (2\pi)^4\delta^{(4)}(p_f - p_i)i\mathcal{M}_{fi} = \delta_{CD,AB} - 2\pi i\delta(s_{AB} - s_{CD})T_{{\rm LC}fi}\,, \tag{9}$$

with $s_{AB} = \frac{m_A^2 + p_{A\perp}^2}{p_A^+/P^+} + \frac{m_B^2 + p_{B\perp}^2}{p_B^+/P^+}$. The T matrix for scattering of composites is given by [4,13]

$$T_{{\rm LC}fi} = P^+ T_{fi} = \langle C|V_D^\dagger \frac{1}{s_{AB} + i\epsilon - H_{\rm LC}} V_B|A\rangle + \langle C|DV_B|A\rangle\,. \tag{10}$$

Here $|A\rangle$ and $|C\rangle$ are composite-particle eigenstates of the light-cone Hamiltonian $H_{\rm LC}$, and the operator V_B is defined by

$$V_B = [H_{\rm LC}, B^\dagger] - \frac{m_B^2 + p_{B\perp}^2}{p_B^+/P^+} B^\dagger \,, \tag{11}$$

with $B^\dagger$ the creation operator for the B particle, *i.e.* $|B\rangle = B^\dagger|0\rangle$. This construction generalizes one presented some time ago by Wick [13]. Details can be found in Ref. [4]. Given numerical solutions for the composite-particle eigenstates, obtained with DLCQ, the most difficult remaining task is the estimation of the matrix element of $(s + i\epsilon - H_{\rm LC})^{-1}$. For this type of matrix element, the recursion method of Haydock [14] has worked well. A nonrelativistic application is described in [4]; an application to the field-theoretic model studied in [2] is in progress.

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